

Fig. 1 Comparison of two methods for computation of Z-matrix of rectangular planar segments.

tion of input impedance  $Z_{in}$ . The number of terms summed up are indicated on two curves. It may be noted that, if the algorithm proposed in this paper is used, the number of terms needed for 1 percent accuracy is 10, while for 0.1 percent accuracy the number of terms needed is 35.

### III. CONCLUSIONS

A method for faster computations of Z-matrices for rectangular segments in planar microstrip circuits has been presented. As seen by the sample comparison presented, the proposed method yields a dramatic increase in computational efficiency.

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## On Gain-Bandwidth Product for Distributed Amplifiers

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**Abstract**—Contours of constant gain-bandwidth product as a function of the gate and drain attenuation factors are presented. Design tradeoffs are established. It is shown that only one design achieves maximum gain-bandwidth, although many possible choices approach this maximum. The curves also lead to the specification of active device parameters when circuit requirements are known.

### I. INTRODUCTION

In a previous paper by Beyer *et al.* [1], a graphical design technique was presented which included a curve showing maximum gain-bandwidth product. It will be shown in this paper that the previously presented curve is actually a portion of a more general series of contours of varying gain-bandwidth product. We also show that for the choice of a particular MESFET, there exists only one design for a distributed amplifier that offers maximum gain-bandwidth, however a large number of designs may closely approach this maximum.

In designing microwave-distributed amplifiers, it is usually desirable to attempt to achieve the maximum gain-bandwidth product allowed by the choice of a particular transistor. Because of the nonlinear relationship in a distributed amplifier between gain and bandwidth, their product is influenced by circuit parameters in a complex manner. In this paper, we present a set of curves that augment the graphical techniques presented in [1] and show design tradeoffs clearly.

### II. GAIN-BANDWIDTH CONTOURS

Expressing 18 of [1] in terms of  $-3$ -dB bandwidth yields

$$A_0 f_{-3\text{ dB}} = 4KX_{-3\text{ dB}} f_{\text{max}} \quad (1)$$

where

$A$  = dc gain

$f_{-3\text{ dB}}$  = half-power frequency

$$K = \sqrt{ab} e^{-b}$$

$X_{-3\text{ dB}} = f_{-3\text{ dB}}/f_c$  bandwidth normalized to the line cutoff frequency

$f_{\text{max}}$  = MESFET maximum frequency of oscillation.

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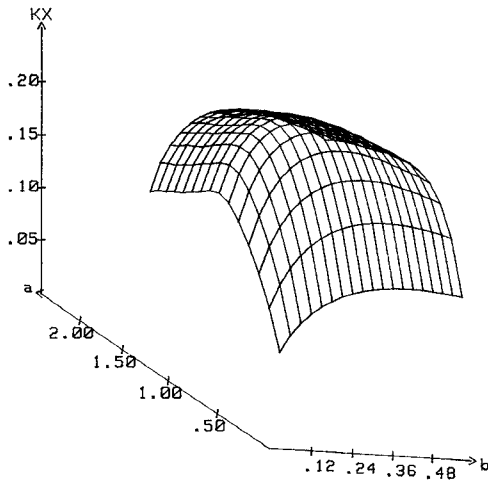


Fig. 1. Normalized gain-bandwidth versus normalized attenuation.

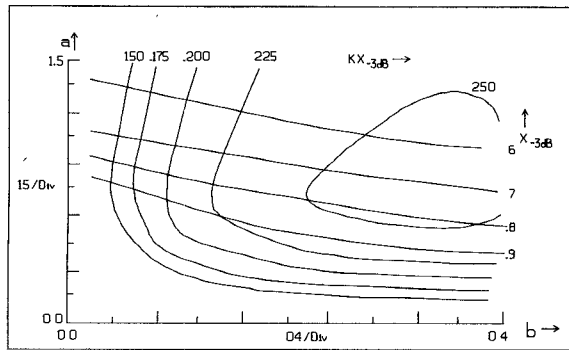


Fig. 2. Normalized gain-bandwidth contours.

The factors  $K$  and  $X_{-3dB}$  are functions of the gate and drain line attenuations as shown in [1], and since the  $A_0 f_{-3dB}$  product cannot exceed  $f_{max}$ , the  $KX_{-3dB}$  product in (1) cannot exceed 0.25. If one plots the  $KX_{-3dB}$  product as a function of the gate and drain line attenuation factors  $a$  and  $b$ , respectively, as defined in [1], the result is the surface shown in Fig. 1. One can easily see that the previously presented maximum gain-bandwidth product curve lies near the crest of the  $KX$  product surface of Fig. 1. Furthermore, one can see that there is a single maximum for gain-bandwidth at  $a = 0.75$ ,  $b = 0.32$ , and that the predicted maximum has a value of 0.255. That this value is about 2 percent greater than the expected value of 0.25 is a result of the approximations used for the attenuation of the gate and drain transmission lines in the equations used to arrive at  $X_{-3dB}$  in (1). When the data are presented in the form of Fig. 2, all of the design techniques of [1] can be readily applied.

The curves presented in Figs. 1 and 2 have their foundation in the behavior of  $X_{-3dB}$  and  $K$  in the  $a-b$  plane. Microwave MESFET-distributed amplifier behavior is controlled by gate and drain line attenuation. From the normalized bandwidth curves presented in [1], one can state in general that the practical bandwidth increases as one approaches the  $b$ -axis and decreases as the  $a$ -axis is approached. Furthermore, normalized gain increases as one approaches the  $a$ -axis and decreases as the  $b$ -axis is approached. Thus, we have the product of an increasing function and a decreasing function along each axis and this type of functional behavior gives rise to the contours of Figs. 1 and 2.

Because these contours close, there is only one design for the distributed amplifier which yields the maximum gain-bandwidth product; all others are suboptimal. This is not to say that it is

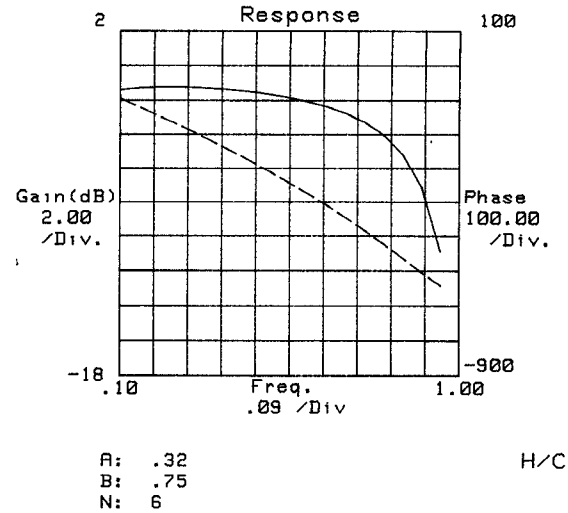


Fig. 3. Distributed amplifier-frequency response.

mandatory to design for maximum gain-bandwidth. If the potential gain-bandwidth exceeds the amplifier requirements, the designer can choose a suboptimal design which meets the stipulated gain and bandwidth specifications. The frequency response of a distributed amplifier showing relative gain versus frequency normalized to the line cutoff for an optimal design ( $a = 0.75$ ,  $b = 0.32$ ) amplifier is shown in Fig. 3. The  $-3$ -dB bandwidth is about 0.74.

### III. DEVICE DESIGN

If one were to consider designing an active device for use specifically in distributed amplifiers, these curves take on greater significance. Equation 15 in [1] gives the dc gain of a distributed amplifier as

$$A_0 = \frac{g_m}{2} (R_{01} R_{02})^{1/2} \frac{\sinh(b)}{\sinh(b/N)} e^{-b}. \quad (2)$$

Using the definition of  $\sinh(b)$  and the small argument approximation to  $\sinh(b/N)$ , we find

$$N \cdot g_m \approx \frac{A_0}{\sqrt{R_{01} R_{02}}} \frac{4b}{1 - e^{-2b}}. \quad (3)$$

Using 12 and 13 in [1] which define the  $a$  and  $b$  factors, we find

$$R_g = a \cdot R_{01} / N \quad (4)$$

$$R_{ds} = N \cdot \frac{R_{02}}{4b} \quad (5)$$

as the gate and drain parasitic resistances. From the definitions of  $R_{01}$ ,  $R_{02}$ , and  $f_c$ , one can show

$$C_{gs} = \frac{1}{\pi f_c R_{01}} \quad (6)$$

$$C_{ds} = \frac{1}{\pi f_c R_{02}}. \quad (7)$$

In deriving (6) and (7), it was assumed that the phase-velocity matching constraint of equal gate- and drain-transmission line cutoff frequencies is met, although small deviations from exact matching may be beneficial to overall performance [2].

The use of these equations may be demonstrated by designing a hypothetical amplifier. The amplifier is required to have 15-dB low-frequency gain, a 20-GHz  $-3$ -dB bandwidth, and 50- $\Omega$  input

and output impedances. The  $a$  and  $b$  coefficients are chosen to achieve the maximum gain-bandwidth which sets  $a = 0.75$  and  $b = 0.32$ .

At this point, the choice of device  $g_m$  must be made. It must be chosen with some care as it determines the number of devices  $N$ , which also affects the parasitic resistances  $R_g$  and  $R_{ds}$ . Using (3), we choose to set  $N = 8$  and solve for  $g_m = 38$  mmhos. From Fig. 2, at  $(a, b) = (0.75, 0.32)$ ,  $X \approx 0.74$ . Since we require  $f_{-3dB} = 20$  GHz, this sets  $f_c = 27$  GHz. Based upon these values, the remaining values are

$$C_{gs} = 0.23 \text{ pF}$$

$$R_g = 4.7 \Omega$$

$$C_{ds} = 0.235 \text{ pF}$$

$$R_{ds} = 312 \Omega.$$

The transistor is now completely specified. The fact that  $C_{gs}$  and  $C_{ds}$  are equal results because the input and output line impedances have been set equal to one another.

### III. CONCLUSION

We have shown that the maximum normalized gain-bandwidth curve in [1] is a small portion of the more general gain-bandwidth contours. There is a maximum which corresponds to the optimum design of a distributed amplifier. Constraint-free design equations for transistors specifically intended for use in distributed amplifiers were also presented.

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## Dispersion of Picosecond Pulses in Coplanar Transmission Lines

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**Abstract**—The dispersion of coplanar-type transmission lines has been extended to the terahertz regime to examine the distortion of picosecond electrical pulses. Dispersion of coplanar waveguides is compared to equivalent microstrip lines. Agreement with available experimental data is demonstrated for coplanar strips. An approximate dispersion formula for coplanar waveguides is also reported for CAD applications.

### I. INTRODUCTION

Picosecond electrical pulses generated by opto-electronic switches [1] have several hundred gigahertz bandwidth and are therefore much dispersed within a few millimeters of travel, even on high-frequency transmission lines such as microstrips and coplanar waveguides. Dispersion characteristics have been in-

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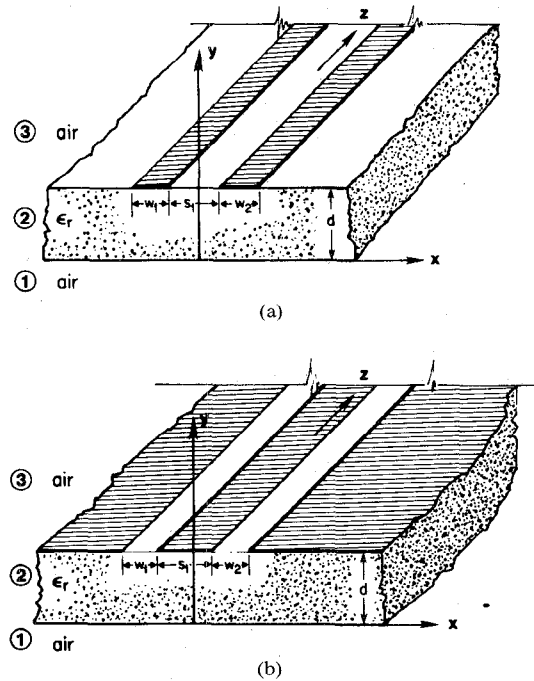


Fig. 1. Two examples of a coplanar-type transmission lines. (a) Coplanar strips (CPS). (b) Coplanar waveguide (CPW).

vestigated thoroughly for the popular microstrip line [2], but published data for coplanar lines are usually limited to about 50 GHz. In previous papers [3], [4], we examined the dispersion of picosecond pulses in microstrip lines. In this paper, we extend the dispersion relation of coplanar-type transmission lines into the terahertz regime and use the result to compute distortion of picosecond pulses propagating in such lines.

### II. THEORY

The spectral domain analysis method used here was first proposed for slot lines by Itoh and Mittra [5] and later extended by Knorr and Kuchler [6] to coupled slots and coplanar strips. For the purpose of clarity, the main steps of the analysis are briefly reiterated. Typical coplanar transmission lines consist of two or more metal strips separated by slots on a dielectric substrate (Fig. 1). The problem is to find the solution to the wave equation in an inhomogeneous medium with inhomogeneous boundary conditions. Since the metal discontinuities lead to difficulties in defining the boundary conditions in the transverse direction, the scalar potentials  $\varphi^{e,h}(x, y)$  are transformed into the Fourier domain. Thus, the Helmholtz wave equation is converted to an ordinary differential equation whose solutions are given by:

$$\varphi_1(\alpha, y) = A(\alpha) e^{-\gamma_1(y-d)} \quad (1)$$

$$\varphi_2(\alpha, y) = B(\alpha) \sinh(\gamma_2 y) + C(\alpha) \cosh(\gamma_2 y) \quad (2)$$

$$\varphi_3(\alpha, y) = D(\alpha) e^{\gamma_3 y} \quad (3)$$

where  $\gamma_i^2 = \alpha^2 + \beta^2 - \omega^2 \mu_0 \epsilon_0 \epsilon_i$ ,  $i = 1, 2, 3$  define the regions,  $\alpha$  is the transform variable corresponding to the  $x$ -variation, and  $\beta$  is the propagation constant in the longitudinal direction. Using the continuity conditions at  $y = 0$

$$E_{z2} = E_{z3}; \quad E_{x2} = E_{x3}; \quad H_{z2} = H_{z3}; \quad H_{x2} = H_{x3} \quad (4)$$